The Inverse Square Law of Light

The relationship between distance and brightness,
and how astronomers measure
distances to far away objects

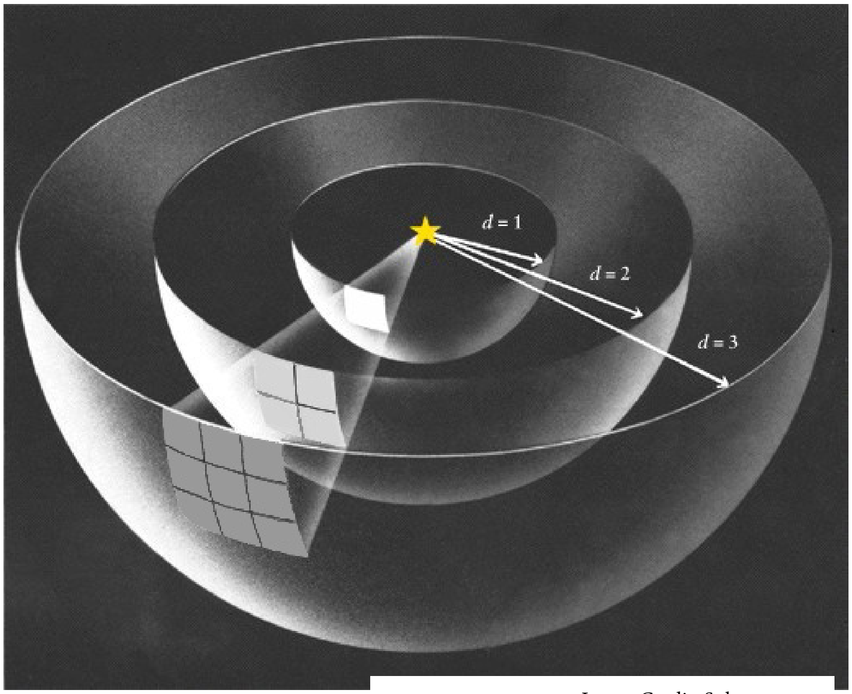


Image Credit: Splung.com

Overview:

We all know that a light, such as a candle or a streetlight, looks dimmer the farther away from it we get. This activity gives an easy way for students to measure the relationship between distance and brightness. Once students discover the relationship, they can begin to understand how astronomers use this knowledge to determine the distances to stars and far away galaxies.

Grade Level:

This lesson is intended for grades 5-12. (See standards correlation in Appendix A.)

Objectives:

Students will be able to:

- demonstrate that the brightness of a source of light is a function of the inverse square of its distance.
- understand how the brightness of light could be used to measure distances, even to stars and far away galaxies.

Connection to Other Activities: This activity is the third in a package of three activities called "Rulers of the Universe". It can be done on its own, but it can also be prefaced by the other two activities: 'Size and Scale of the Universe' and 'Parallax'. They can be found at http://wise.ssl.berkeley.edu/education_class.html

Time:

Prep Time: For each shade box, 15 minutes if using a MiniMaglite™, 30 minutes or less if you make the light source
Part I: 45- 60 minutes
Part II: 45-60 minutes

Acknowledgments:

Nancy Alima Ali, Dr. Bryan Mendez and Kyle Fricke developed and produced these lessons using funding from the Education and Public Outreach program of the NASA Wide-field Infrared Survey Explorer Mission while at the Center for Science Education at the Space Sciences Laboratory (CSE@SSL) at the University of California in Berkeley. Karin Hauck, Darlene Yan and Leitha Thrall assisted with the testing of the lesson. We would also like to thank the Astrophysics Educator Ambassadors and teachers attending the "Cool Astronomy" workshop at the American Museum of Natural History for their feedback on the lesson. This lesson was initially adapted from an Exploratorium "Science Snacks" activity for use in the Universe Forum at the Harvard Smithsonian Center for Astrophysics.

"The Inverse Square Law of Light" lesson is part of a series of lessons exploring the "Size and Scale of the Universe". Additional lessons, including ones using WISE data, may be found at the WISE website http://wise.ssl.berkeley.edu/index.html.

The Wide-field Infrared Survey Explorer, or WISE, is a NASA-funded Explorer mission that surveyed the entire sky in infrared light. WISE continues to provide a vast storehouse of knowledge about the Solar System, the Milky Way, and the Universe.

Materials:

I. For each student:
   - 1 student data table (Copy master provided in Appendix B)
   - 1 sheet graph paper (Copy master provided in Appendix C)
   - Calculator (or brain)

II. For each shade box (1 shade box per group of 2-4 students):
- 3 sheets heavy black poster board, size 10.5” x 13”
- 2 sheets white card stock, size 10.5” x 13”
- 1 sheet white card stock, size 11” x 2”
- 1 sheet graph paper (copy master provided in this guide)

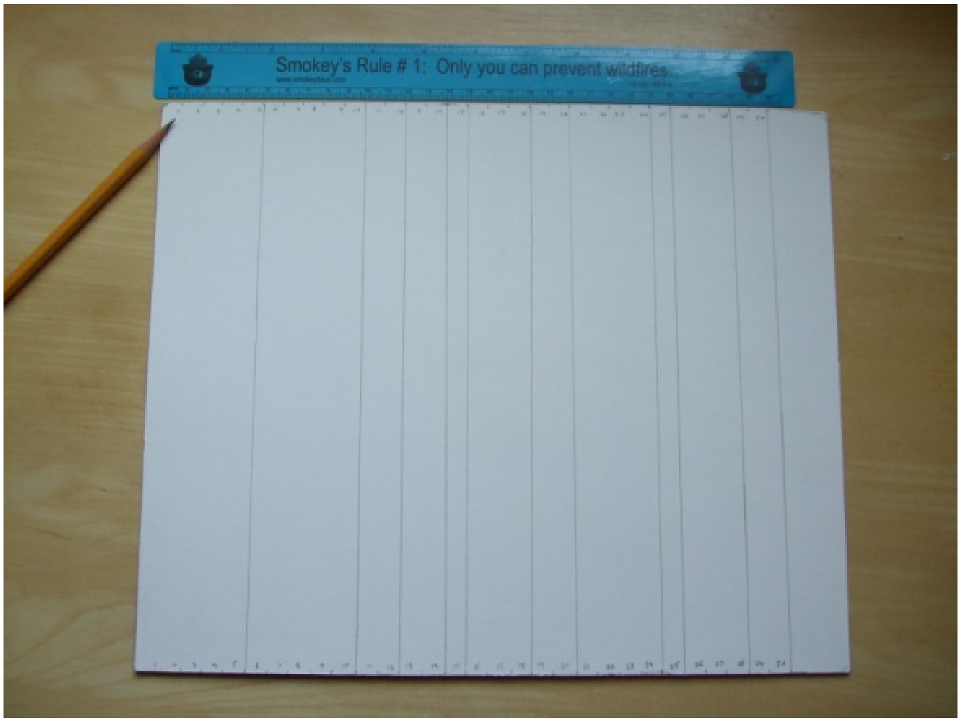
III. A Mini-Maglite™ flashlight (1 per shade box/group)
Do NOT use a regular flashlight (or a MicroMaglite™) as a substitute. A point source of light is required for this activity. A cheaper alternative to using a Mini-Maglite™ is to create your own light source using a miniature light bulb that has two leads, two batteries (either AAA, AA, C, or D) and alligator clips to connect them. Using the alligator clips, wire the bulb in series with the batteries.

IV. For making the shade boxes: an Exacto knife or scissors, transparent tape, ruler with centimeter markings, pencil, stapler

Getting Ready:

Note #1:
This activity requires one shade box with MiniMaglite™ flashlight for each group of 2-4 students. You might consider having volunteers assist in constructing the shade boxes or have students construct them during class time. If it is not possible to get enough materials for all students, consider using this activity in station format with students using the materials in rotation.

1. Construct the Box:
   - On both long sides of the white 10.5” x 13” card stock, use a ruler and pencil to measure and mark small notches for each centimeter. Be sure to start measuring right at the end of the card. Number the notches on both sides. Draw straight lines joining the notches at 4, 5, 10, 12, 14, 15, 18, 20, 24, 25, 28 and 30 centimeters.

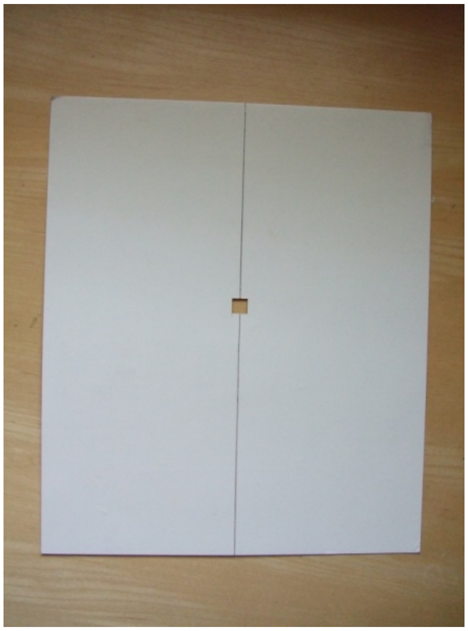


- Tape the graph paper to one sheet of black 10.5" x 13" poster board. Tape this to a second sheet of black poster board, joining the two along the long sides. The two boards should be like an open book with the graph paper on the left side. Stand upright to create one corner of the box.

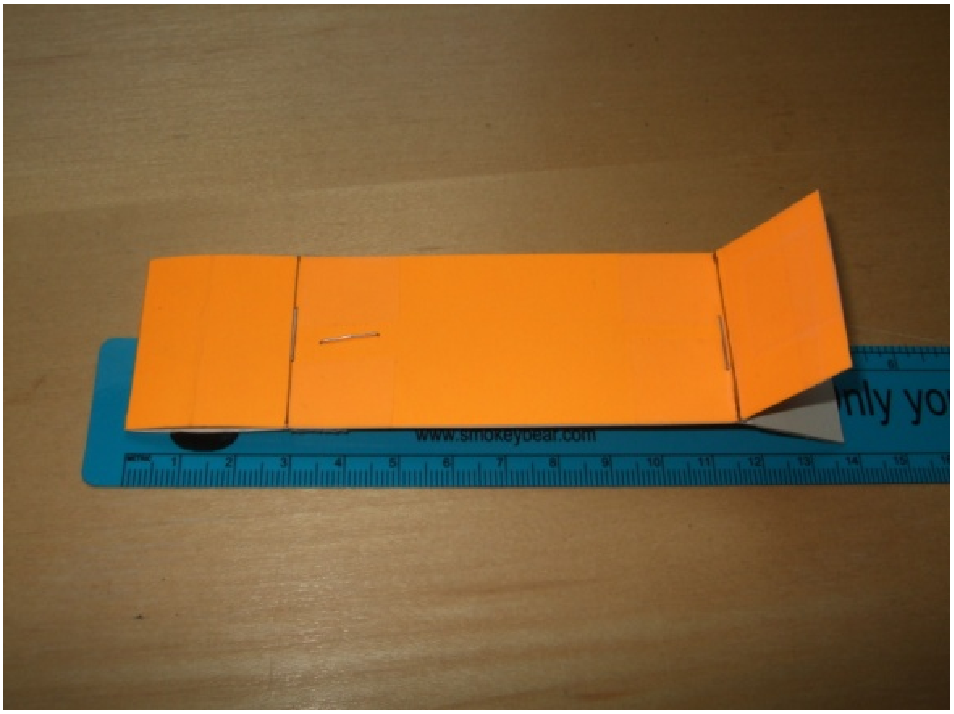
- To create the bottom of the box, tape the white card with cm lines to the piece of black poster board that has the graph paper. Make sure the centimeter markings are visible with the 1 cm mark closest to the graph paper.

- To create the top of the box, balance the third sheet of black poster board on top of the sides of the box.
  NOTE: Each completed box will have only four pieces (top and bottom, back and one side.

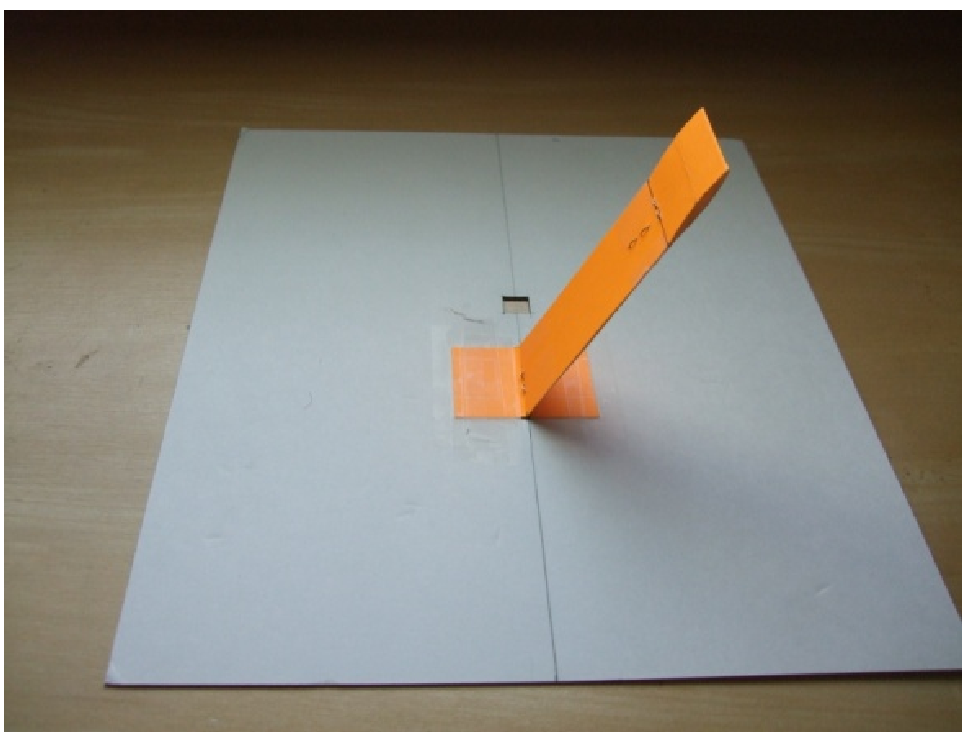
2. Construct the Light Window:
    - Now, draw a line lengthwise through the center of the remaining piece of white 10.5" x 13" card stock. Measure and cut a 1 x 1 cm square hole in the poster board centered on this line with the bottom of the square 16 cm from one end of the card stock. It is important that the window is located at a height of 16 cm because that is how tall the MiniMagliteTM is when standing up in candle mode. (If using an Exacto knife, place old cardboard underneath the card stock to protect the table.

3. Construct the Holder for the MiniMaglite™
    - Fold the 11" x 2" card stock in half at the 5.5" (14 cm) point.
    - Starting from the fold, measure 3 cm towards the open ends. Draw a line along the width (short side) at this point. Staple along this line.
    - Starting from the open ends, measure 2 cm towards the crease. Draw a line along the width (short side) at the 2 cm point. Staple along this line.
    - Fold the open ends outward along the 2 cm line, to create a butterfly crease.
    - NOTE: It is important to measure accurately when constructing the holder since it will ensure that the light is kept at a constant 10 cm distance from the window.



4. Attach the Holder to the Window Card
   - Tape or staple the holder's ends on the center line of the poster board that has the 1 x 1 cm hole. It generally works best if the holder is near the window.



5. Convert the MiniMagliteTM into Candle Mode
    - Unscrew the flashlight head to expose the bulb.
    - Place the head face down on a stable flat surface
    - Placing the flashlight barrel into the head. Putting the MiniMagliteTM barrel into the head is important for stability and also to ensure the light bulb is the correct height for the activity.

    NOTE: Avoid touching the light bulb as it may be hot.

Image Credit: www.maglite.com

6. Putting it all Together
   - Slip the MiniMaglite™ into the holder. Place the holder opposite the graph paper in the box. The square of light made when the Mini-Maglite™ shines through this hole will shine on the graph paper.

Procedure:

Part I - Gathering the Data

Engage

Tell students to imagine they are standing a sidewalk at night and see a motorcycle coming towards them on the street. Ask them how the light from the motorcycle's headlight would change as it comes closer. Would the light become brighter or dimmer as the motorcycle got closer? (As the distance decreases, the light becomes brighter.)

Tell students that astronomers use these same concepts to estimate the distances of stars. Explain that in this activity, students will measure the relationship between distance and brightness.

Explore

1. Divide students into pairs and give each pair an assembled shade box, window card, data table and graph paper. Have students set up their shade box with the MiniMagliteTM or miniature bulb in the window card.

   Turn off the classroom lights and have students place the bulb at a distance of 10 cm from the graph paper. (The window card should be pressed up against the graph paper.) Students count how many squares on the graph paper are lit then record the distance and number of illuminated squares in the first two columns of the data table.

2. Students put the bulb different distances from the graph paper (e.g. 14, 15, 18, 20, 24, 25, 28, 30 cm), and count how many squares on the graph paper are lit at each distance. Remind students to make sure to measure the distance from the bulb, not the window card. Students record distances and number of squares illuminated in the first two columns of the data table.

3. Students measure the size of the squares in the graph paper to determine the area of each square. If you use the graph paper provided with this activity the sides should be 1/2 cm, and thus each square has an area of 1/4 cm2. Students calculate the area illuminated at each distance measured, and record it in the third column of the data table.

4. To complete the fourth column of the data table, students will need to calculate the relative brightness for each distance using the formula B/B0 = 1/A. Before having students do the calculations, discuss with them the meaning behind the formula.

   Remind students that what we are interested in knowing is how distance affects the amount of light that falls on each square. The amount of light received per area is called brightness. The amount of light given off by the bulb and passing through the hole in the card always remains constant. This is called luminosity.

   So, what we want to calculate is the brightness relative to some standard brightness (say the brightness of the bulb on the graph paper at 10 cm). Let's look at the relationships mathematically. We call brightness B, area A, and the luminosity L, and we can write the following:

   $B = \frac{L}{A}$ for any distance, and $B_0 = \frac{L}{A_0}$ for your standard distance (10 cm).

   So the relative brightness is $\frac{B}{B_0} = \frac{A_0}{A}$ (L cancels out because it is the same for both).

   With a standard distance of 10 cm, the area illuminated was 1 cm2. So, $A_0 = 1$ and we have:

$$\boldsymbol{\frac{B}{B_0} = \frac{1}{A}}$$

   Next, have students calculate the relative brightness for each distance, and record it in the last column of the data table.

   NOTE: If you are using graph paper that has different size squares, the same formula will work.

   NOTE: In the formula, "B/B0" represents the relative brightness and since it is a ratio, it is dimensionless

Explain

Part II – Graphing, Analyzing & Discussing the Data

1. Using the information from the data table, students make a graph of the relative brightness as a function of distance. The x axis represents distance (in cm) and the y axis represents apparent brightness.

2. After students have completed their graphs, discuss the results as a class. In examining your graph, can you determine how brightness depends on distance? Is it directly proportional, inversely proportional, proportional to the inverse square, etc.? Have students come up with a statement that explains the relationship between brightness and distance. Show the students the completed graph that measures the measured relative brightness versus the theoretical brightness for the inverse square law of light.

3. Discuss with students how astronomers use the inverse square law of light to measure distances to stars or galaxies. (See Discussion Notes, below).

Elaborate

Discussion Notes:

The light from the Mini-Maglite™ spreads out equally in all directions. As the distance from the bulb to the graph paper increases, the same amount of light spreads over a larger and larger area and the light reaching each square becomes correspondingly less bright.

Adjust the distance from the bulb to the graph paper to 10 cm. At this distance, the graph paper touches the card. A 1 cm2 area will be illuminated. When the graph paper is moved 20 cm from the card, 4 cm2 will be illuminated on the graph paper. When the graph paper is moved 30 cm from the card, 9 cm2 will be illuminated, and so on. The area illuminated will increase as the square of the distance.

The brightness of light is the power (energy per second) per area. Since the energy that comes through the hole you cut is constant but spreads out over a larger area, the brightness (or intensity) of light decreases. Since the area increases as the square of the distance, the brightness of the light must decrease as the inverse square of the distance. Thus, brightness follows the inverse-square law.

If you had two light bulbs and knew that they both give off the same amount of light (same luminosity/power), then you could calculate the relative distance between the two of them simply by measuring their relative brightness. If you also knew what the luminosity/power of the bulbs was, you would then be able to determine the distance to both bulbs. Or, if you knew the distance to one of the bulbs you could determine the distance to the other one.

This is how astronomers use the inverse square law of light to measure distances to stars or galaxies. They find stars that are the same kind (same size and temperature) and, therefore, have the same luminosity. They measure the brightness of the stars and can determine distances if they know either what the luminosity of the stars is or the actual distance to one of the stars by some other method.

Cepheid variable stars are particularly useful in determining astronomical distances. Cepheids are stars whose brightness increases and decreases in a regular period of time. Because the relationship between brightness and period is standard, if the variability period is known then the brightness can be inferred. Once the brightness of the star is known, its distance can be calculated by comparing it to another Cepheid star. Thus, Cepheid variables act as the "standard candles" of astronomical distances.

More information about Cepheid Variables can be found at the following websites:

http://starchild.gsfc.nasa.gov/docs/StarChild/shadow/cepheids.html
http://outreach.atnf.csiro.au/education/senior/astrophysics/variable_cepheids.html
http://www.spitzer.caltech.edu/news/1243-ssc2011-01-Cosmology-Standard-Candle-Not-So-Standard-After-All

Example Data:

Distance From Bulb (cm)	Number of Squares Illuminated	Area Illuminated (cm^2)	Relative Brightness
10	4	1.00	1.00
14	8.4	2.10	0.48
15	9.3	2.33	0.43
18	13.3	3.33	0.30
20	16.4	4.10	0.24
24	23.5	5.88	0.17
25	26	6.50	0.15
28	34.8	8.70	0.115
30	36.6	9.15	0.109

Note:

The graph on the next page plots these data as points and also plots a line representing how the relative brightness should theoretically depend on distance: B/B0 = 100/d2

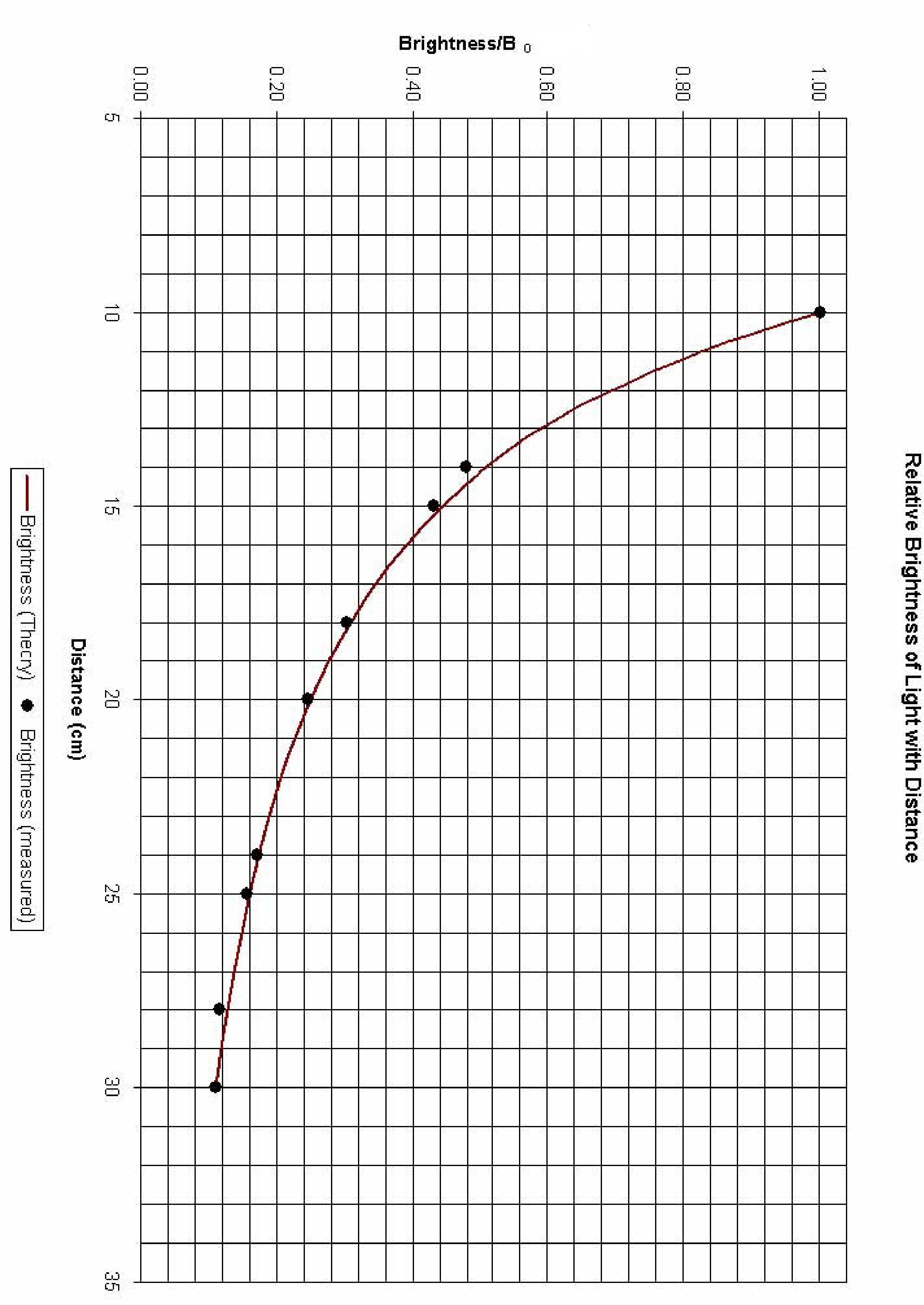
This is derived by assuming that the area illuminated is proportional to the square of the distance and solving for the constant of proportionality…

A = kd2

For d = 10 cm, A = 1 cm2

Thus, k = 1/100

Sample Graph – Measured & Theoretical Brightness:



Evaluate

Assessment:

Formative

During the lesson, check to make sure that students have completed the data table and graph correctly. See the “Example Data” table for detail.

Summative

At the end of the lesson, ask students to complete the following questions (in order of increasing difficulty).

1. Using your completed graph for reference, if the relative brightness is 0.60, what would be the distance of the light source?
   (Answer: Approximately 13 cm)

2. Using your completed graph for reference, if the distance of the light source increased to 40 cm, how would its relative brightness change?
   (Answer: The relative brightness would decrease and move closer to 0.)

3. Two stars (A & B) have the same relative brightness. Star A is at a distance of 100 light-years. Star B is at a distance of 400 light-years. Which star is more is more luminous and by how much? Show your work.
   (Answer: Star B is 16 times more luminous. B=L/4*Pi*d2 so BA/BB = 1 = (LA/LB)*(dB/dA)2 so LB/LA = (400/100)2 = 16)

4. Two stars (C & D) are the same type of star and have the same luminosity. Star D appears to have only 1.2% of the brightness of Star C. Star C is known to be 20 light-years away. How far away is Star D? Show your work.
   (Answer: BD/BC = 0.012 = (LD/LC)*(dC/dD)2 so dC/dD = SQRT(0.012) and dD = 20/SQRT(0.012) = 182 light-years)

Appendix A: Standards Correlation

Grades 6-8:

AAAS Benchmarks for Science Literacy

- Graphs can show a variety of possible relationships between two variables. (9B/M3)
- Simulations are often useful in modeling events and processes. (11B/M4)
- Organize information in simple tables and graphs and identify relationships they reveal. (12D/M1)

National Science Education Standards

Content Standard A: Science as Inquiry – Abilities necessary to do scientific inquiry:

- Use appropriate tools and techniques to gather, analyze and interpret data.
- Use mathematics in all aspects of scientific inquiry.

National Council of Teachers of Mathematics Standards

Algebra:

- Use symbolic algebra to represent situations and to solve problems, especially those that involve linear relationships.
- Model and solve contextualized problems using various representations, such as graphs, tables, and equations
- Use graphs to analyze the nature of changes in quantities in linear relationships

Grades 9-12:

AAAS Benchmarks for Science Literacy

- As energy spreads out, whether by conduction, convection, or radiation, the total amount of energy stays the same. However, since it is spread out, less can be done with it. (4E/H)

National Science Education Standards

Content Standard A: Science as Inquiry – Understandings about Scientific Inquiry

National Council of Teachers of Mathemathics Standards

Algebra

- Use symbolic algebra to represent and explain mathematical relationships
- Draw reasonable conclusions about a situation being modeled
- Approximate and interpret rates of change from graphical and numerical data

Appendix B: Student Data Table

Name: ____________________

Date: ____________________

Distance From Bulb (cm)	# of Squares Illuminated	Area Illuminated (cm^2)	Relative Brightness